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An analytical error for the mean radial length method of strain analysis

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Abstract

An analytical expression for the error associated with estimating the axial ratio (R_s) and orientation (ϕ_s) of the finite strain ellipse with the mean radial length method of strain analysis is presented. Analytical errors are computationally efficient and compare excellently with errors calculated by the computationally intensive bootstrap approach.

Keywords: Strain analysis; Mean radial length; Analytical error; Confidence interval

1. Introduction

Recently, Mulchrone et al. (2003) introduced another method for strain analysis using the mean radial length (MRL) of strain markers. The method is similar to previous methods such as those by Shimamoto and Ikeda (1976) and Wheeler (1984), all of which are mathematical expressions of the original idea by Ramsay (1967, pp. 216–221). Along with a plethora of other methods, Mulchrone and Roy Choudhury (2004) have demonstrated that MRL can be applied to populations of objects of any shape, thus ensuring the broad applicability of MRL. This result is contingent on fitting ellipses to marker objects using methods incorporating the moment data of the object shape. Mulchrone et al. (2003) estimated the error associated with MRL by applying the versatile, but computationally intensive bootstrap method (Efron, 1979). In order to reduce the computational overhead and complexity of implementation associated with the bootstrap approach to error analysis, an analytical expression for the error associated with the MRL method is derived below. Although computational overhead may no longer be a significant issue, an analytical solution is certainly easier to implement for structural geologists without programming skills. A comparison is made between errors calculated with the expression derived here and the bootstrap for a set of natural data. The method of derivation is general enough to be applied to other areas of structural geology and is therefore of general interest.

2. Error analysis

MRL was derived by Mulchrone et al. (2003) by assuming that the initial orientations of the long axes are from a uniform distribution and that the distribution of initial axial ratios is axially symmetric. The method is based on the conceptually simple fact that averaging the parameters of an initial ellipse distribution (i.e. the unstrained state) defines a circle, so that in the strained state the averaged parameters define an ellipse, namely the strain ellipse. For each ellipse, where the ith ellipse has axial ratio R_i and orientation ϕ_i, the following values are calculated

$$m_i = \frac{1}{2}\left(R_i - \frac{1}{R_i}\right) \tag{1}$$

$$p_i = \frac{1}{2}\left(R_i + \frac{1}{R_i}\right) \tag{2}$$

and subsequently the following weighted means may be determined

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$$q_s = \frac{\sum_{i=1}^{n} w_i p_i}{\sum_{i=1}^{n} w_i} \tag{3}$$

$$t_s = \frac{\sum_{i=1}^{n} w_i m_i \cos(2\phi_i)}{\sum_{i=1}^{n} w_i} \tag{4}$$

$$u_s = \frac{\sum_{i=1}^{n} w_i m_i \sin(2\phi_i)}{\sum_{i=1}^{n} w_i} \tag{5}$$

where n is the number of marker objects. Note that this expression differs from that given in Mulchrone and Roy Choudhury (2004) because they assumed that $\sum_{i=1}^{n} w_i = 1$, whereas here a more general form is used. Note that setting $w_i = 1$ for all i results in the original expressions derived by Mulchrone et al. (2003). The strain ellipse parameters (axial ratio R_s and orientation ϕ_s) can then be calculated as follows:

$$\tan(2\phi_s) = \frac{u_s}{t_s} \tag{6}$$

$$R_s = \sqrt{\frac{q_s \cos(2\phi_s) + t_s}{q_s \cos(2\phi_s) - t_s}} \tag{7}$$

To derive analytical expressions for the errors associated with ϕ_s and R_s, expressions are required for the weighted variance of the weighted means in Eqs. (3)–(5). It is important to note that the weighted variance of the weighted mean is sought and not the weighted variance of the population, hence the following expressions have been divided by n (see for example Devore (1995, p. 230)). The variances $\sigma_{q_s}^2$, $\sigma_{t_s}^2$ and $\sigma_{u_s}^2$ for q_s, t_s and u_s, respectively, are given by (Bevington, 1969, p. 73)

$$\sigma_{q_s}^2 = \frac{\sum_{i=1}^{n} w_i (p_i - q_s)^2}{(n-1)\sum_{i=1}^{n} w_i} \tag{8}$$

$$\sigma_{t_s}^2 = \frac{\sum_{i=1}^{n} w_i (m_i \cos(2\phi_i) - t_s)^2}{(n-1)\sum_{i=1}^{n} w_i} \tag{9}$$

$$\sigma_{u_s}^2 = \frac{\sum_{i=1}^{n} w_i (m_i \sin(2\phi_i) - u_s)^2}{(n-1)\sum_{i=1}^{n} w_i} \tag{10}$$

and can be readily calculated.

An estimate for the error of ϕ_s and R_s is obtained using a Taylor series expansion for the variance (Hahn and Shapiro, 1967, pp. 230–232; Bevington and Robinson, 1992, pp. 41–43). In general, suppose there is a function f of estimated parameters (p_1, p_2,...) with known variances $(\sigma_{p_1}^2, \sigma_{p_2}^2, \ldots)$ such that

$$z = f(p_1, p_2, \ldots) \tag{11}$$

it can be shown by taking the Taylor series approximation to f at the parameter means $(\bar{p}_1, \bar{p}_2, \ldots)$ that the variance of z is approximated by

$$\sigma_z^2 \approx \sigma_{p_1}^2 \left(\frac{\partial f}{\partial p_1}\right)^2 + \sigma_{p_2}^2 \left(\frac{\partial f}{\partial p_2}\right)^2 + \cdots \tag{12}$$

where σ_z^2 is the variance of z. It is worth noting that higher order terms have been omitted, although this is usually considered to be a reasonable approximation in most cases. In any case, the validity of this approximation for the MRL is cross-checked using an independent bootstrap estimation. Taking the formulas for ϕ_s and R_s above, and substituting the value for ϕ_s from Eq. (6) into Eq. (7), and for conciseness let

$$\psi = q_s + t_s\sqrt{1 + \frac{u_s^2}{t_s^2}} \qquad \gamma = q_s - t_s\sqrt{1 + \frac{u_s^2}{t_s^2}}$$

$$\beta = u_s^2 + t_s^2$$

we have:

$$\phi_s = \frac{1}{2}\tan^{-1}\left(\frac{u_s}{t_s}\right) \tag{13}$$

$$R_s = \sqrt{\frac{\psi}{\gamma}} \tag{14}$$

There are two cases to be considered when applying Eq. (14) and deriving expressions for error. The values of both q_s and $\sqrt{1 + (u_s^2/t_s^2)}$ are always positive, therefore if $t_s \geq 0$, Eq. (14) must be used as shown, otherwise if $t_s < 0$, the reciprocal of Eq. (14) must be used to ensure that $R_s > 1$ in every case.

From Eq. (12), the estimated variance associated with ϕ_s (i.e. $\hat{\sigma}_{\phi_s}^2$) is:

$$\hat{\sigma}_{\phi_s}^2 \approx \frac{\sigma_{u_s}^2 t_s^2 + \sigma_{t_s}^2 u_s^2}{4\beta^2} \tag{15}$$

Calculating $\hat{\sigma}_{R_s}^2$, the variance of R_s, is slightly more involved because depending on the sign of t_s a different equation is used. Therefore:

$$\hat{\sigma}_{R_s}^2 \approx \frac{\sigma_{q_s}^2 \beta^2 + q_s^2(\sigma_{t_s}^2 t_s^2 + \sigma_{u_s}^2 u_s^2)}{\beta\psi\gamma^3}, \quad \text{if } t_s \geq 0 \tag{16}$$

$$\hat{\sigma}_{R_s}^2 \approx \frac{\sigma_{q_s}^2 \beta^2 + q_s^2(\sigma_{t_s}^2 t_s^2 + \sigma_{u_s}^2 u_s^2)}{\beta\psi^3\gamma}, \quad \text{if } t_s < 0 \tag{17}$$

However, from application to natural and randomly generated data, it has been found (see Section 3) that higher order terms need to be included in the estimation of $\hat{\sigma}_{R_s}^2$ for satisfactory results. Eq. (12) with higher order terms included is given by (Bevington and Robinson, 1992, pp. 41–43)

$$\sigma_z^2 \approx \sigma_{p_1}^2 \left(\frac{\partial f}{\partial p_1}\right)^2 + \sigma_{p_2}^2 \left(\frac{\partial f}{\partial p_2}\right)^2 + 2\sigma_{p_1p_2}^2 \left(\frac{\partial f}{\partial p_1}\frac{\partial f}{\partial p_2}\right) + \cdots \tag{18}$$

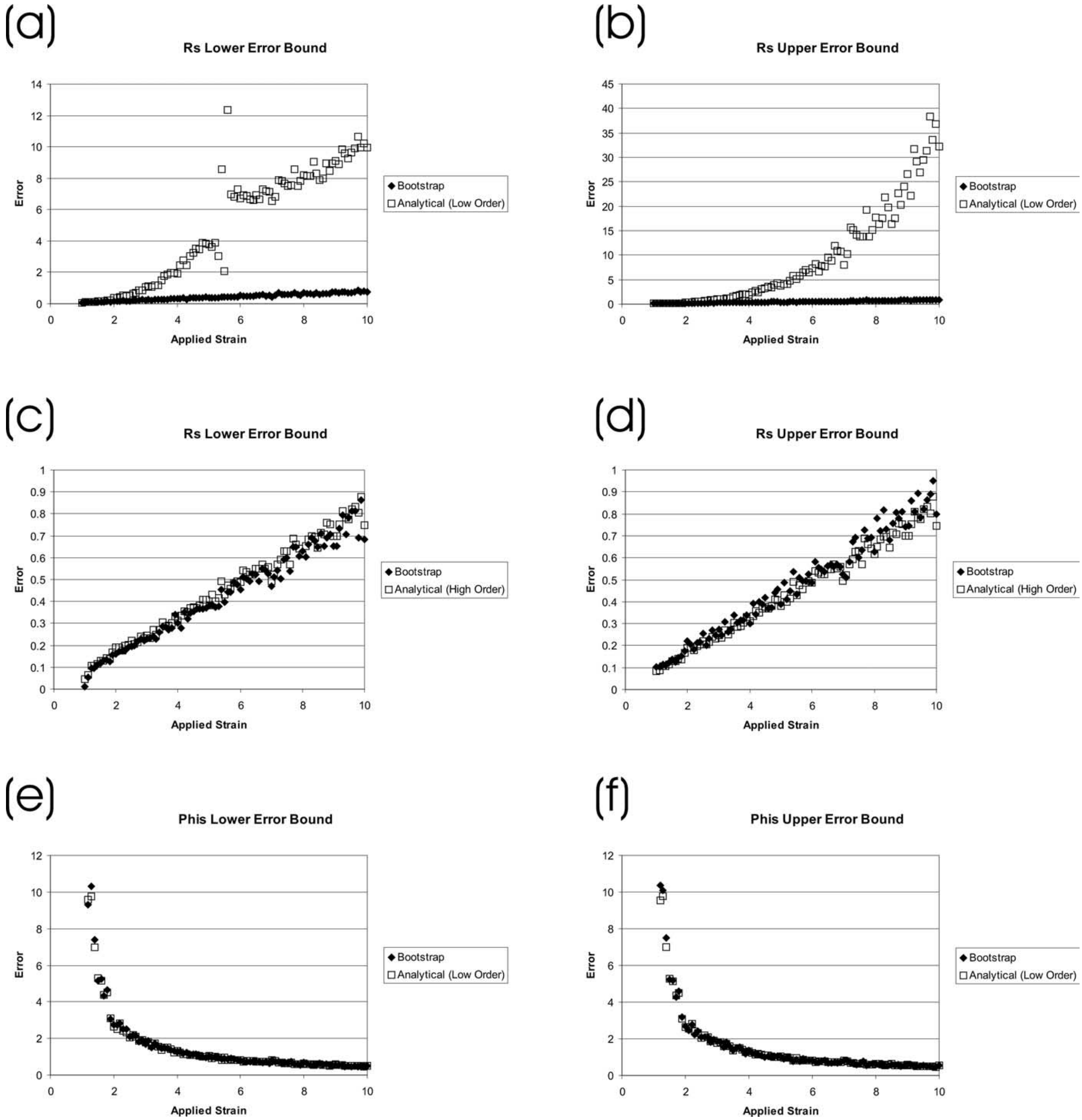

Fig. 1. Results of simulation study to compare analytical and bootstrap errors. Variation of error with increasing applied strain for (a) lower error bound on R_s and (b) upper error bound on R_s calculated using the low-order expressions (Eqs. (16) and (17)); (c) lower error bound on R_s and (d) upper error bound on R_s calculated using the high-order expressions (Eqs. (19) and (20)); (e) lower error bound on ϕ_s and (f) upper error bound on ϕ_s.

so that the variance of R_s is given by

$$\hat{\sigma}^2_{R_s} \approx \frac{2q_s(\sigma^2_{t_s q_s} t_s + \sigma^2_{u_s q_s} u_s)\beta - \sigma^2_{q_s}\beta^2 - q_s^2(\sigma^2_{t_s} t_s^2 + u_s(2\sigma^2_{u_s t_s} t_s + \sigma^2_{u_s} u_s))}{\beta\psi\gamma^3} \tag{19}$$

if $t_s \geq 0$ and

$$\hat{\sigma}^2_{R_s} \approx \frac{2q_s(\sigma^2_{t_s q_s} t_s + \sigma^2_{u_s q_s} u_s)\beta - \sigma^2_{q_s}\beta^2 - q_s^2(\sigma^2_{t_s} t_s^2 + u_s(2\sigma^2_{u_s t_s} t_s + \sigma^2_{u_s} u_s))}{\beta\psi^3\gamma} \tag{20}$$

if $t_s < 0$, where $\sigma^2_{t_s q_s}$ is the covariance of t_s and q_s, and $\sigma^2_{u_s q_s}$, $\sigma^2_{u_s t_s}$ are similarly defined. These quantities can be

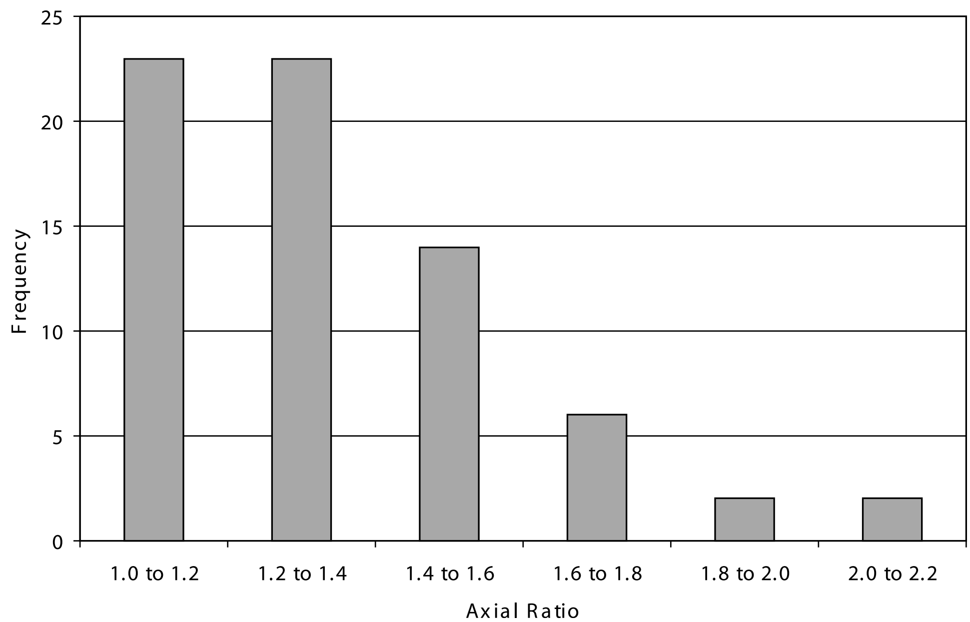

Fig. 2. Frequency distribution of the finite strain axial ratios of the natural data.

calculated as follows:

$$\sigma^2_{t_s q_s} = \frac{\sum_{i=1}^{n} w_i (m_i \cos(2\phi_i) - t_s)(p_i - q_s)}{(n-1)\sum_{i=1}^{n} w_i} \quad (21)$$

$$\sigma^2_{u_s t_s} = \frac{\sum_{i=1}^{n} w_i (m_i \sin(2\phi_i) - u_s)(m_i \cos(2\phi_i) - t_s)}{(n-1)\sum_{i=1}^{n} w_i} \quad (22)$$

$$\sigma^2_{u_s q_s} = \frac{\sum_{i=1}^{n} w_i (m_i \sin(2\phi_i) - u_s)(p_i - q_s)}{(n-1)\sum_{i=1}^{n} w_i} \quad (23)$$

The $100(1-\alpha)\%$ confidence interval for each parameter is then given by

$$\phi_s - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2_{\phi_s}} \le \phi_s \le \phi_s + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2_{\phi_s}} \quad (24)$$

$$R_s - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2_{R_s}} \le R_s \le R_s + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2_{R_s}} \quad (25)$$

where $t_{\alpha/2,n-p}$ is the Student's t-test statistic, n is the number of data and p is the number of parameters (i.e. 2 in the present case). For large data sets (usually $n>30$; see Devore, 1995, p. 293), the Student's t-test can be replaced by the standard normal (represented by $z_{\alpha/2}$), which at the 95% confidence level has a value of approximately 1.96. Meere and Mulchrone (2003) demonstrate that data sets with $n\approx150$ are a minimum for strain analysis, so that the normal approximation is perfectly valid for these cases.

3. Comparison with bootstrap

In order to validate the above approach, a comparison was made between bootstrap errors calculated at the 95% confidence interval and those calculated using the analytical method derived above for both computer-generated and natural data. Datasets containing 150 data each were generated by randomly selecting an axial ratio between 1 and 10 and a long axis orientation between 0 and 180° using a uniform distribution (see Mulchrone et al., 2003; Meere and Mulchrone, 2003 for further details). Each random dataset was strained to a known finite strain ranging from 1.0 to 10 in 0.1 steps and an estimate of ϕ_s and R_s was obtained, as well as an estimate of the associated errors using the analytical approach derived above and the bootstrap. Results are presented in Fig. 1. It is clear that the lower order approximation for errors associated with R_s (i.e. Eqs. (16) and (17)) breaks down for applied finite strains greater than around 2.0 (see Fig. 1a and b). By contrast, the higher order approximation works well up to applied finite strains of at least 10.0 and is the recommended method. The lower order approximation for errors associated with ϕ_s closely match with those calculated by the bootstrap and therefore the lower order approximation is recommended in this case.

Quartz clast shape data was collected from 68 thin sections of 23 sandstone samples from the Variscides of southwest Ireland (Meere, 1995). Three sections were examined per sample cut parallel to the fabric, perpendicular to the regional fold axes and parallel to the fold axes, but perpendicular to the fabric. Fifty clasts were measured from each thin section and re-analysed using MRL. Although 50 measurements are below the level (150) recommended by Meere and Mulchrone (2003) this just means that the confidence intervals are larger than ideal. Three samples of sandstone, oolite and conglomerate, each providing 150 measurements, were also analysed. The natural samples represent a variety of strain conditions from low to medium

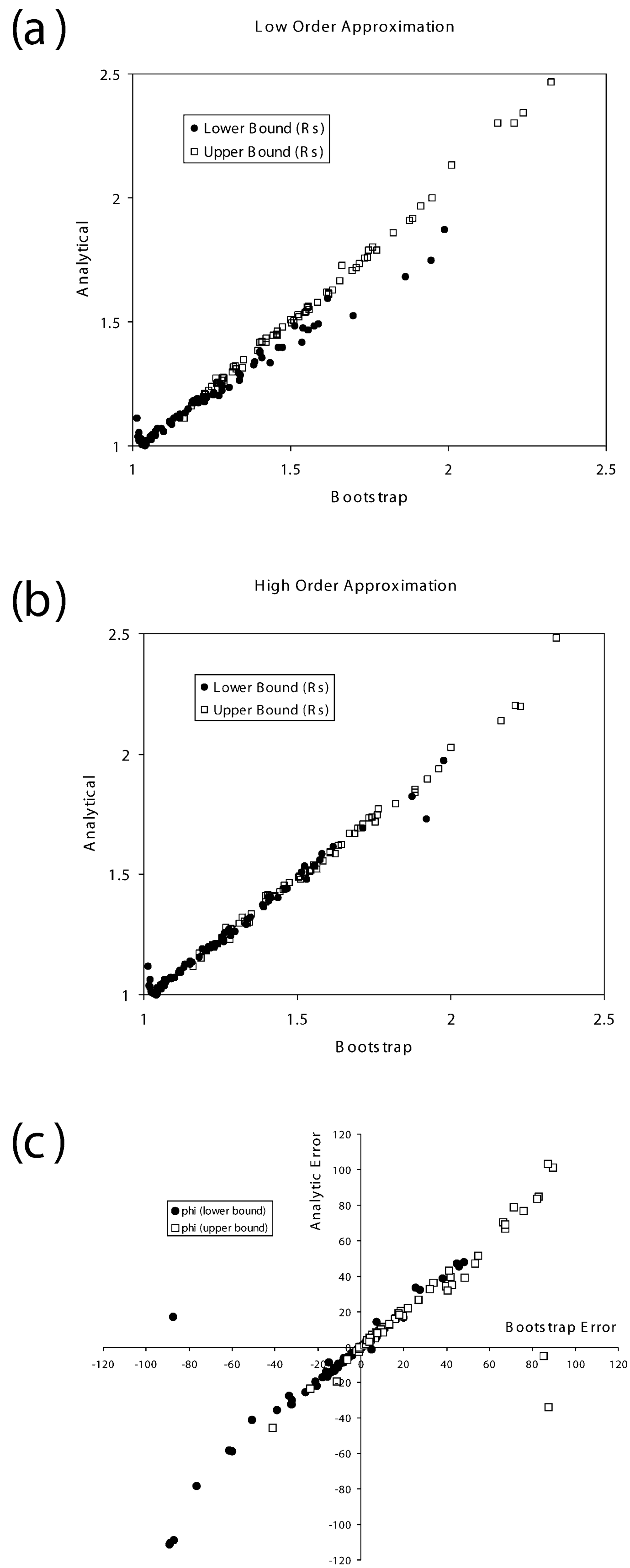

Fig. 3. Plot of bootstrap errors versus analytical errors associated with estimation of (a) R_s with the low-order formula (Eqs. (16) and (17)), (b) R_s with the high-order formula (Eqs. (19) and (20)) and (c) ϕ_s.

Table 1
Detailed results of analysis of natural data

Sample	Lithology	*n*	Bootstrap						Analytical					
			Rsl	Rscalc	Rsu	Phil	Phicalc	Phiu	Rsl	Rscalc	Rsu	Phil	Phicalc	Phiu
10AB	Sandstone	50	1.24	1.35	1.50	−9.99	−1.52	6.53	1.21	1.35	1.49	−9.56	−1.52	6.52
10AC	Sandstone	50	1.41	1.55	1.70	−5.13	−0.29	4.77	1.40	1.55	1.69	−5.31	−0.29	4.73
10BC	Sandstone	50	1.05	1.14	1.25	−89.44	85.16	89.28	1.04	1.14	1.24	69.00	85.16	101.31
11AB	Sandstone	50	1.29	1.44	1.63	3.91	12.07	18.99	1.26	1.44	1.62	3.89	12.07	20.24
11AC	Sandstone	50	1.44	1.60	1.82	−12.11	−5.67	−0.28	1.40	1.60	1.79	−11.46	−5.67	0.11
11BC	Sandstone	50	1.02	1.10	1.23	−71.35	56.61	80.66	1.01	1.10	1.20	28.51	56.61	84.71
12AB	Sandstone	50	1.07	1.17	1.31	−0.07	16.58	37.16	1.04	1.17	1.30	−1.17	16.58	34.33
12AC	Sandstone	50	1.57	1.73	1.92	−7.42	−1.26	5.53	1.56	1.73	1.90	−8.03	−1.26	5.50
12BC	Sandstone	49	1.04	1.12	1.24	26.74	55.23	76.53	1.03	1.12	1.21	33.57	55.23	76.89
13AB	Sandstone	50	1.18	1.28	1.42	45.38	56.28	67.41	1.16	1.28	1.41	45.67	56.28	66.90
13AC	Sandstone	50	1.02	1.07	1.22	−85.78	−56.93	85.49	1.07	1.07	1.21	−108.89	−56.93	−4.96
13BC	Sandstone	51	1.10	1.19	1.33	44.99	65.14	83.34	1.07	1.19	1.30	46.93	65.14	83.35
14AB	Sandstone	49	1.13	1.27	1.40	−6.51	5.70	19.92	1.13	1.27	1.41	−5.87	5.70	17.27
14AC	Sandstone	51	1.28	1.40	1.58	−13.36	−4.42	4.73	1.24	1.40	1.56	−13.95	−4.42	5.11
14BC	Sandstone	50	1.04	1.09	1.18	3.47	34.43	64.27	1.00	1.09	1.17	−1.52	34.43	70.38
15AB	Sandstone	50	1.39	1.53	1.71	−2.97	3.09	8.68	1.37	1.53	1.69	−2.86	3.09	9.03
15AC	Sandstone	51	1.30	1.43	1.61	−25.10	−16.16	−6.11	1.26	1.43	1.59	−25.42	−16.16	−6.91
15BC	Sandstone	50	1.02	1.08	1.20	−52.16	−3.14	49.12	1.03	1.08	1.18	−41.18	−3.14	34.90
16AB	Sandstone	50	1.12	1.20	1.34	−20.12	−7.49	5.64	1.10	1.20	1.30	−21.88	−7.49	6.89
16AC	Sandstone	51	1.08	1.17	1.27	−77.30	−61.78	−42.08	1.06	1.17	1.28	−78.18	−61.78	−45.38
16BC	Sandstone	51	1.16	1.24	1.35	−20.63	−9.94	−1.13	1.14	1.24	1.34	−19.56	−9.94	−0.32
17AB	Sandstone	50	1.41	1.53	1.69	−14.66	−8.33	−2.36	1.39	1.53	1.67	−14.50	−8.33	−2.17
17AC	Sandstone	50	1.21	1.33	1.47	−1.21	10.37	21.75	1.20	1.33	1.47	−0.92	10.37	21.65
17BC	Sandstone	50	1.26	1.35	1.51	1.33	10.16	17.53	1.22	1.35	1.48	1.35	10.16	18.97
18AB	Sandstone	50	1.09	1.24	1.40	2.05	16.76	38.44	1.07	1.24	1.42	1.50	16.76	32.02
18AC	Sandstone	51	1.15	1.28	1.44	−31.89	−17.38	−0.46	1.13	1.28	1.43	−32.16	−17.38	−2.61
18BC	Sandstone	50	1.02	1.07	1.20	−62.69	−9.46	48.71	1.03	1.07	1.18	−58.16	−9.46	39.23
19AB	Sandstone	50	1.46	1.58	1.75	−7.70	−0.87	4.91	1.44	1.58	1.72	−7.25	−0.87	5.51
19AC	Sandstone	50	1.19	1.30	1.41	−6.61	4.60	16.18	1.19	1.30	1.41	−7.19	4.60	16.39
19BC	Sandstone	49	1.06	1.15	1.27	−9.15	14.95	33.86	1.04	1.15	1.25	−6.36	14.95	36.26
1AB	Sandstone	50	1.03	1.10	1.23	−14.81	19.34	51.15	1.00	1.10	1.21	−8.44	19.34	47.12
1AC	Sandstone	50	1.20	1.35	1.52	−4.16	6.52	16.08	1.18	1.35	1.52	−3.33	6.52	16.37
1BC	Sandstone	50	1.04	1.13	1.27	−33.81	−0.45	29.19	1.00	1.13	1.25	−27.62	−0.45	26.73
20AB	Sandstone	51	1.41	1.59	1.76	−14.26	−7.62	−0.57	1.41	1.59	1.77	−14.35	−7.62	−0.88
20AC	Sandstone	50	1.26	1.37	1.54	−12.91	−2.20	9.29	1.23	1.37	1.52	−13.13	−2.20	8.73
20BC	Sandstone	52	1.07	1.16	1.28	−33.08	−12.46	5.37	1.05	1.16	1.27	−29.75	−12.46	4.84
21AB	Sandstone	49	1.23	1.35	1.51	−10.61	−3.05	4.22	1.21	1.35	1.49	−10.26	−3.05	4.17
21AC	Sandstone	50	1.22	1.32	1.45	−9.05	0.13	10.33	1.20	1.32	1.44	−9.21	0.13	9.46
21BC	Sandstone	50	1.22	1.32	1.46	−6.63	2.40	11.52	1.20	1.32	1.44	−6.48	2.40	11.28
22AB	Sandstone	50	1.34	1.45	1.61	18.16	25.27	32.23	1.31	1.45	1.59	17.88	25.27	32.66
22AC	Sandstone	50	1.07	1.18	1.34	−38.90	−17.90	−0.93	1.06	1.18	1.30	−35.50	−17.90	−0.29
22BC	Sandstone	49	1.06	1.16	1.29	6.27	22.49	43.78	1.05	1.16	1.27	5.53	22.49	39.46
23AB	Sandstone	50	1.26	1.39	1.56	0.44	9.24	17.43	1.24	1.39	1.54	0.20	9.24	18.28
23AC	Sandstone	50	1.35	1.47	1.64	−4.10	3.29	11.09	1.32	1.47	1.63	−4.46	3.29	11.04

(*continued on next page*)

Table 1 (*continued*)

Sample	Lithology	*n*	Bootstrap						Analytical					
			Rsl	Rscalc	Rsu	Phil	Phicalc	Phiu	Rsl	Rscalc	Rsu	Phil	Phicalc	Phiu
23BC	Sandstone	49	1.33	1.44	1.62	−12.56	−6.28	1.71	1.29	1.44	1.59	−13.53	−6.28	0.97
2AB	Sandstone	50	1.47	1.59	1.76	−0.29	4.85	9.02	1.44	1.59	1.75	−0.01	4.85	9.71
2AC	Sandstone	50	1.28	1.40	1.56	−15.35	−7.11	2.23	1.27	1.40	1.52	−16.78	−7.11	2.56
2BC	Sandstone	50	1.03	1.07	1.19	−87.19	60.03	87.34	1.01	1.07	1.15	16.98	60.03	103.09
3AB	Sandstone	55	1.15	1.33	1.53	−4.69	6.14	15.66	1.14	1.33	1.51	−3.47	6.14	15.74
3AC	Sandstone	50	1.23	1.36	1.54	−5.64	3.50	12.82	1.20	1.36	1.51	−5.77	3.50	12.78
3BC	Sandstone	49	1.01	1.01	1.16	−86.09	−56.06	86.01	1.12	1.01	1.12	−577.36	−56.06	465.24
4AB	Sandstone	50	1.92	2.11	2.34	−31.74	−27.93	−23.47	1.73	2.11	2.48	−32.29	−27.93	−23.58
4AC	Sandstone	50	1.52	1.74	1.96	−7.91	−2.55	2.66	1.53	1.74	1.94	−7.88	−2.55	2.77
4BC	Sandstone	50	1.07	1.15	1.28	25.74	55.34	71.78	1.06	1.15	1.23	32.12	55.34	78.56
5AB	Sandstone	50	1.55	1.69	1.88	−7.84	−3.38	1.95	1.54	1.69	1.84	−8.67	−3.38	1.91
5AC	Sandstone	50	1.26	1.47	1.67	−7.66	0.32	10.59	1.26	1.47	1.67	−7.70	0.32	8.34
5BC	Sandstone	50	1.02	1.07	1.19	−87.95	−72.10	88.16	1.04	1.07	1.18	−110.21	−72.10	−34.00
6AB	Sandstone	50	1.58	1.81	2.00	−1.89	3.49	6.98	1.59	1.81	2.03	−1.30	3.49	8.29
6AC	Sandstone	50	1.87	2.01	2.23	−2.16	2.65	7.40	1.83	2.01	2.20	−2.42	2.65	7.73
6BC	Sandstone	50	1.13	1.18	1.27	38.13	53.93	66.95	1.12	1.18	1.25	38.62	53.93	69.24
7AB	Sandstone	50	1.06	1.17	1.32	−62.54	−39.05	−12.31	1.03	1.17	1.32	−58.66	−39.05	−19.45
7AC	Sandstone	50	1.11	1.24	1.40	20.22	34.05	53.84	1.09	1.24	1.39	16.56	34.05	51.54
7BC	Sandstone	50	1.21	1.30	1.43	−18.01	−6.60	3.08	1.19	1.30	1.41	−16.98	−6.60	3.78
8AB	Sandstone	50	1.71	1.91	2.16	−5.94	0.05	3.91	1.69	1.91	2.14	−5.39	0.05	5.50
8AC	Sandstone	54	1.12	1.27	1.46	5.74	28.51	40.14	1.09	1.27	1.46	14.08	28.51	42.95
8BC	Sandstone	50	1.28	1.40	1.55	−12.16	−3.62	4.91	1.26	1.40	1.54	−12.10	−3.62	4.87
9AB	Sandstone	52	1.53	1.67	1.88	2.20	7.18	13.44	1.48	1.67	1.86	1.36	7.18	12.99
9AC	Sandstone	50	1.39	1.56	1.74	−10.73	−4.08	4.30	1.38	1.56	1.74	−11.30	−4.08	3.15
9BC	Sandstone	50	1.09	1.17	1.28	−16.89	2.17	17.49	1.07	1.17	1.27	−13.84	2.17	18.19
oolite	Oolite	158	1.62	1.68	1.73	45.57	47.84	50.17	1.62	1.68	1.74	45.65	47.84	50.04
sst	Sandstone	151	1.51	1.61	1.71	−10.29	−6.09	−2.93	1.51	1.61	1.71	−9.76	−6.09	−2.42
conglom	Comglomerate	138	1.98	2.09	2.21	5.15	7.45	9.82	1.97	2.09	2.20	5.11	7.45	9.80

n is the number of data, Rsl is the lower bound on R_s, Rscalc is the calculated value of R_s, Rsu is the upper bound on R_s, phil is the lower bound on ϕ_s, phicalc is the calculated value of ϕ_s and phiu is the upper bound on ϕ_s.

(i.e. $R_s = 1.0$–2.2, see Fig. 2). The results are presented in Fig. 3 and Table 1. There is a striking correlation between the bootstrap and analytical errors for both R_s and ϕ_s, illustrating that both methods produce almost identical error bounds except that the analytical method is a lot less computationally expensive and easier to implement. Note that there is a slight underestimation of the lower error bound evident in Fig. 3a for the low-order analytical method (i.e. using Eqs. (16) and (17)). This is to be expected in the light of the simulation results presented above. It is clear that the higher order error approximation (i.e. using Eqs. (19) and (20)) gives a much better correlation, as predicted by the simulation. In the case of the errors for ϕ_s there are three outliers departing from the general pattern. This can be readily explained by low finite strain values measured for these samples. At low finite strains, a large spread of possible values for ϕ_s are produced by the bootstrap calculations so that the bootstrap records an error interval from approximately -90 to 90° independent of the calculated value of ϕ_s.

4. Conclusions

An analytical expression for the error associated with estimating R_s and ϕ_s with the MRL method (Mulchrone et al., 2003) has been presented. Analytical errors are computationally efficient and compare excellently with errors calculated by the computationally intensive bootstrap approach.

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